Max Planck

The Universe in the Light of Modern Physics

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by

DR. MAX PLANCK

Professor of Theoretical Physics at the
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Translated by

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The present work is a translation of two books, "Das Weltbild der neuen Physik" and "Physikalische Gesetzlichkeit im Lichte neuerer Forschung," published by Joh. A. Barth in Leipzig. The two works have been run into one, the second here commencing on p. 58.

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THE UNIVERSE IN THE LIGHT OF MODERN PHYSICS

§ 1

Physics is an exact Science and hence depends upon measurement, while all measurement itself requires sense-perception. Consequently all the ideas employed in Physics are derived from the world of sense-perception. It follows from this that the laws of Physics ultimately refer to events in the world of the senses; and in view of this fact many scientists and philosophers tend to the belief that at bottom Physics is concerned exclusively with this particular world. What they have in mind, of course, is the world of man's senses. On this view, for example, what is called an "Object" in ordinary parlance is, when regarded from the standpoint of Physics, simply a combination of different sense-data localized in one place. It is worth pointing out that this view cannot be refuted by logic, since logic itself is unable to lead us beyond the confines of our own senses; it cannot even compel one to admit the independent existence of others outside oneself.

In Physics, however, as in every other science, common sense alone is not supreme; there must also be a place for Reason. Further, the mere absence of logical contradiction does not necessarily imply that everything is reasonable. Now reason tells us that if we turn our back upon a so-called object and cease to attend to it, the object still continues to exist. Reason tells us further that both the individual man and mankind as a whole, together with the entire world which we apprehend through our senses, is no more than a tiny fragment in the vastness of Nature, whose laws are in no way affected by any human brain. On the contrary, they existed long before there was any life on earth, and will continue to exist long after the last physicist has perished.

It is considerations of this kind, and not any logical argument, that compel us to assume the existence of another world of reality behind the world of the senses; a world which has existence independent of man, and which can only be perceived indirectly through the medium of the world of the senses, and by means of certain symbols which our senses allow us to apprehend. It is as though we were compelled to contemplate a certain object in which we are interested through spectacles of whose optical properties we were entirely ignorant.

If the reader experiences difficulty in following this argument, and finds himself unable to accept the idea of a real world which at the same time is expressly asserted to lie beyond our senses, we might point out that there is a vast difference between a physical theory complete in every detail, and the construction of such a theory. In the former case the content of the theory can be analysed exactly, so that it is possible to prove at every point that the notions which we apply to the world of sense are adequate to the formulation of this theory; in the latter case we must develop a theory from a number of individual measurements. The second problem is very much more difficult, while the history of Physics shows that whenever it has been solved, this has been done on the assumption of a real world independent of our senses; and it seems reasonably certain that this will continue to be the case in the future.

But besides the world of sense and the real world, there is also a third world which must be carefully distinguished from these:—this is the world of Physics. It differs from the two others because it is a deliberate hypothesis put forward by a finite human mind; and as such, it is subject to change and to a kind of evolution. Thus the function of this world of Physics may be described in two ways, according as it is related to the real world, or to the world of the senses. In the first case the problem is to apprehend the real world as completely as possible; in the second, to describe the world of the senses in the simplest possible terms. There is no need, however, to assign superior merit to either of these formulations, since each of them, taken by itself alone, is incomplete and unsatisfactory. On the one hand, the real world cannot be apprehended directly at all; while on the other no definite answer is possible to the question:—Which is the simplest description of a given number of interdependent sense-perceptions? In the history of Physics it has happened more than once that, of two descriptions, one was for a time considered the more complicated but was later discovered to be the simpler of the two.

The essential point therefore is that these two formulations of the problem, when practically applied, shall be complementary to each other and not contradictory. The first is an indispensable aid to the groping imagination of the investigator, supplying him with ideas without which his work remains unfruitful; the second provides him with a firm foundation of facts. In actual practice individual physicists are influenced in their investigations by their personal preference for metaphysical, or for positivist, ideas. But besides the metaphysicians and the positivists there is a third group of students who investigate the world from the physical point of view. They differ from the first two groups in being interested not so much in the relation between the world of physics on the one hand, and the real world and the world of sense-data on the other, as in the internal consistency and logical structure of the world of physics. These men form the axiomatic school, whose activity is as necessary and useful as is that of the others. At the same time, they are equally exposed to the danger of specialization which, in their case, would lead to a barren formalism taking the place of a fuller understanding of the world of Physics. For as soon as contact with reality has been lost, physical law ceases to be felt as the relation between a number of magnitudes which have been ascertained independently of one another, and becomes a mere definition by which one of these magnitudes is derived from the others. In this method there is a particular attraction, due to the fact that a physical magnitude can be defined far more exactly by means of an equation than by means of measurement. But at the same time, this method amounts to a renunciation of the true meaning of magnitude; while it must also be remembered that confusion and misunderstanding result when the same name is retained in order to denote a changed meaning.

We see, then, how physicists are at work in different directions and from different standpoints in elaborating a systematic view of the world of Physics. Nevertheless the aim of all these endeavours is the same, and consists in establishing a law which connects the events of the world of sense with one another and with those of the real world. Naturally, these different tendencies predominated in turn at different stages of history. Whenever the physical world presented a stable appearance, as in the second half of the last century, the metaphysical view tended to predominate, and it was believed that a complete grasp of the real world was relatively near. Conversely, in times of change and insecurity like the present, positivism tends to occupy the foreground; for in such times a careful student will tend to seek support where he can find real security; and this is to be found precisely in the events of the world of the senses.

Now if we consider the different forms which the view of the physical world has taken in the course of history, and if we look for the peculiarities which characterized these changes, two facts will strike us with special force. First, it is plain that when regarded as a whole, all the changes in the different views of the world of Physics do not constitute a rhythmical swing of the pendulum. On the contrary, we find a clear course of evolution making more or less steady progress in a definite direction; progress which is best described by saying that it adds to the content of the world of sense, rendering our knowledge more profound and giving us a firmer grasp of it. The most striking instance of this is found in the practical application of Physics. Not even the most confirmed sceptic can deny that we see and hear at a greater distance and command greater forces and speeds than an earlier generation; while it is equally certain that this progress is an enduring increase of knowledge, which is in no danger of being described as an error and rejected at any future date.

Secondly, it is a very striking fact that the impulse towards simplification and improvement of the world-picture of Physics was due in each instance to some kind of novel observation—that is, to some event in the world of sense. But at the same moment the structure of this physical world consistently moved farther and farther away from the world of sense and lost its former anthropomorphic character. Still further, physical sensations have been progressively eliminated, as for example in physical optics, in which the human eye no longer plays any part at all. Thus the physical world has become progressively more and more abstract; purely formal mathematical operations play a growing part, while qualitative differences tend to be explained more and more by means of quantitative differences.

Now we have already pointed out that the physical view of the world has been continually perfected and also related to the world of sense. If this fact is added to those mentioned in the last paragraph, the result is extraordinarily striking; at first, indeed, it appears completely paradoxical. Of this apparent paradox there is, in my opinion, only one rational explanation. This consists in saying that as the view of the physical world is perfected, it simultaneously recedes from the world of sense; and this process is tantamount to an approach to the world of reality. I have no logical proof on which to base this opinion; it is impossible to demonstrate the existence of the real world by purely rational methods: but at the same time it is equally impossible ever to refute it by logical methods. The final decision must rest upon a common-sense view of the world, and the old maxim still remains true that that world-view is the best which is the most fruitful. Physics would occupy an exceptional position among all the other sciences if it did not recognize the rule that the most far-reaching and valuable results of investigation can only be obtained by following a road leading to a goal which is theoretically unobtainable. This goal is the apprehension of true reality.

§ 2

What changes have taken place in the physical view of the world during the last twenty years? We all know that the changes which have occurred during this period are among the most profound that have ever arisen in the evolution of any science; we also know that the process of change has not yet come to an end. Nevertheless it would appear that in this flux of change certain characteristic forms of the structure of this new world are beginning to crystallize; and it is certainly worth while to attempt a description of these forms, if only in order to suggest certain improvements.

If we compare the old theory with the new, we find that the process of tracing back all qualitative distinctions to quantitative distinctions has been advanced very considerably. All the various chemical phenomena, for example, have now been explained by numerical and spatial relations. According to the modern view there are no more than two ultimate substances, namely positive and negative electricity. Each of these consists of a number of minute particles, similar in nature and with similar charges of an opposite character; the positive particle is called the proton, the negative the electron. Every chemical atom that is electrically neutral consists of a number of protons cohering with one another, and of a similar number of electrons, some of which are firmly fixed to the protons, together with which they form the nucleus of the atom, while the rest revolve around the nucleus.

Thus the Hydrogen atom, the smallest of all, has one proton for nucleus and one electron revolving round the nucleus; while the largest atom, Uranium, contains 238 protons and 238 electrons; but only 92 electrons revolve round the nucleus while the others are fixed in it. Between these two atoms lie all the other elements, with many kinds of different combinations. The chemical properties of an element depend, not on the total number of its protons or electrons, but on the number of revolving electrons, which yield the atomic number of the element.

Apart from this important advance, which is however merely the successful application of an idea first evolved many centuries ago, there are two completely new ideas which distinguish the modern conception of the world from its predecessor; these are the Theory of Relativity, and the Quantum Theory. It is these two ideas which are peculiarly characteristic of the new world of Physics. The fact that they appeared in science almost simultaneously is something of a coincidence; for their content, as well as their practical effect upon the structure of the physical view of the world, are entirely different.

The Theory of Relativity seemed at first to introduce a certain amount of confusion into the traditional ideas of Time and Space; in the long run, however, it has proved to be the completion and culmination of the structure of classical Physics. To express the positive result of the Special Theory of Relativity in a single word, it might be described as the fusion of Time and Space in one unitary concept. It is not, of course, asserted that Time and Space are absolutely similar in nature; their relation resembles that between a real number and an imaginary number, when these are combined together to form the unified concept of a complex number. Looked at in this way, Einstein's work for Physics closely resembles that of Gauss for Mathematics. We might further continue the comparison by saying that the transition from the Special to the General Theory of Relativity is the counterpart in Physics to the transition from linear functions to the general theory of functions in mathematics.

Few comparisons are entirely exact, and the present is no exception to the rule. At the same time it gives a good idea of the fact that the introduction of the Theory of Relativity into the physical view of the world is one of the most important steps towards conferring unity and completeness. This appears clearly in the results of the Theory of Relativity, especially in the fusing of momentum and energy, in the identification of the concept of mass with the concept of energy, of inertial with ponderable mass, and in the reduction of the laws of gravitation to Riemann's geometry.

Brief though these main outlines are, they contain a vast mass of new knowledge. The new ideas mentioned apply to all natural events great and small, beginning with radio-active atoms emanating waves and corpuscles, and ending with the movements of celestial bodies millions of light-years away.

The last word on the Theory of Relativity probably still remains to be said. Surprises may yet await us, especially when we consider that the problem of amalgamating Electrodynamics with Mechanics has not yet been definitely solved. Again, the cosmological implications of the Theory of Relativity have not yet been fully cleared up, the chief reason being that everything depends upon the question whether or not the matter of outer space possesses a finite density; this question has not yet been answered. But whatever reply is eventually given to these questions, nothing will alter the fact that the Principle of Relativity has advanced the classical physical theory to its highest stage of completion, and that its world-view is rounded off in a very satisfactory manner.

This fact will perhaps be a sufficient reason for devoting no more time to the Theory of Relativity; I might also point out that there are many treatises on the Theory adapted to the requirements of readers of every kind.

§ 3

The idea of the universe as thus far described appeared almost perfectly adapted to its purpose; but this state of affairs has suddenly been upset by the Quantum Theory. Here again I shall attempt to describe the characteristic idea of this hypothesis in one word. We may say, then, that its essence consists in the fact that it introduces a new and universal constant, namely the elementary Quantum of Action. It was this constant which, like a new and mysterious messenger from the real world, insisted on turning up in every kind of measurement, and continued to claim a place for itself. On the other hand, it seemed so incompatible with the traditional view of the universe provided by Physics that it eventually destroyed the framework of this older view.

For a time it seemed that a complete collapse of classical Physics was not beyond the bounds of possibility; gradually however it appeared, as had been confidently expected by all who believed in the steady advance of science, that the introduction of the Quantum Theory led not to the destruction of Physics, but to a somewhat profound reconstruction, in the course of which the whole science was rendered more universal. For if the Quantum of Action is assumed to be infinitely small, Quantum Physics becomes merged in classical Physics. In fact the foundations of the structure of classical Physics not only proved unshakable, but actually were rendered firmer through the incorporation of the new ideas. The best course, therefore, will be first to examine the latter.

It will be best to begin by enumerating the essential component features. These are the universal constants, e.g. the gravitational constant, the velocity of light, the mass and charge of electrons and protons. These are perhaps the most tangible symbols of a real world, and they retain their meaning unchanged in the new view of the universe. Further, we may mention the great principles of the conservation of energy and of momentum, which, although they were under suspicion for a time, have eventually emerged unimpaired. It should be emphasized that in this process of transition these principles were proved to be something more than mere definitions, as some members of the Axiomatic School would like to believe. Further, we may mention the main laws of thermodynamics, and especially the second law, which through the introduction of an absolute value for entropy obtained a more exact formulation than it possessed in classical Physics. Lastly we may point to the Principle of Relativity, which has proved itself a reliable and eloquent guide in the new regions of Quantum Physics.

The question may now be asked whether modern Physics differs at all from the older Physics, if all these foundations of classical Physics have remained untouched. It is easy to find an answer to this question by examining the elementary Quantum of Action somewhat more closely. It implies that in principle an equation can be established between energy and frequency; $E = h\nu$.[1] It is this equation which

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[1] In this equation E stands for Energy, and ν for Frequency, that is the number of vibrations per second.

classical Physics utterly fails to explain. The fact itself is so baffling because energy and frequency possess different dimensions; energy is a dynamic magnitude, whereas frequency is a kinematic magnitude. This fact in itself, however, does not contain a contradiction. The Quantum Theory postulates a direct connection between dynamics and kinematics; this connection is due to the fact that the unit of energy, and consequently the unit of mass, are based upon the units of length and of time; thus the connection, so far from being a contradiction, enriches and rounds off the classical theory. There is, nevertheless, a direct contradiction, which renders the new theory incompatible with the classical theory. The following considerations make clear this contradiction. Frequency is a local magnitude, and has a definite meaning only for a certain point in space; this is true alike of mechanical, electric and magnetic vibrations, so that all that is requisite is to observe the point in question for a sufficient time. Energy on the other hand

For example, light vibrations range from about 400 million million per second to about 800 million million.

h represents "Planck's Constant", discovered by the author of this work. It is an unchanging or invariable quantity, and extremely minute, its value being 655 preceded by 26 decimal places. [TRANS.]

is an additive quantity; so that according to the classical theory it is meaningless to speak of energy at a certain point, since it is essential to state the physical system the energy of which is under discussion; just as it is similarly impossible to speak of a definite velocity unless the system be indicated to which velocity is referred. Now we are at liberty to choose whatever physical system we please, either little or great; and consequently the value of the energy is always to a certain extent arbitrary. The difficulty, then, consists in the fact that this arbitrary energy is supposed to be equated with a localized frequency. The gulf between these two concepts should now be clearly apparent: and in order to bridge this gulf a step of fundamental importance must be taken. This step does imply a break with those assumptions which classical Physics has always regarded and employed as axiomatic.

Hitherto it had been believed that the only kind of causality with which any system of Physics could operate was one in which all the events of the physical world—by which, as usual, I mean not the real world but the world-view of Physics—might be explained as being composed of local events taking place in a number of individual and infinitely small parts of Space. It was further believed that each of these elementary events was completely determined by a set of laws without respect to the other events; and was determined exclusively by the local events in its immediate temporal and spatial vicinity. Let us take a concrete instance of sufficiently general application. We will assume that the physical system under consideration consists of a system of particles, moving in a conservative field of force of constant total energy. Then according to classical Physics each individual particle at any time is in a definite state; that is, it has a definite position and a definite velocity, and its movement can be calculated with perfect exactness from its initial state and from the local properties of the field of force in those parts of Space through which the particle passes in the course of its movement. If these data are known, we need know nothing else about the remaining properties of the system of particles under consideration.

In modern mechanics matters are wholly different. According to modern mechanics, merely local relations are no more sufficient for the formulation of the law of motion than would be the microscopic investigation of the different parts of a picture in order to make clear its meaning. On the contrary, it is impossible to obtain an adequate version of the laws for which we are looking, unless the physical system is regarded *as a Whole.* According to modern mechanics, each individual particle of the system, in a certain sense, at any one time, exists simultaneously in every part of the space occupied by the system. This simultaneous existence applies not merely to the field of force with which it is surrounded, but also to its mass and its charge.

Thus we see that nothing less is at stake here than the concept of the particle—the most elementary concept of classical mechanics. We are compelled to give up the earlier essential meaning of this idea; only in a number of special borderline cases can we retain it. But if we pursue the line of thought indicated above, we shall find what it is that we can substitute for the concept of the particle in more general cases.

[*The following brief section may be omitted by readers not interested in the somewhat technical issues, and the subject resumed on p.* 38.]

[The Quantum Theory postulates that an equation subsists between energy and frequency. If this postulate is to have an unambiguous meaning, that is a meaning independent of the particular system to which it is referred, then the Principle of Relativity demands that a momentum vector[1] shall be equivalent to a wave-member vector; in other words, the absolute quantity of the momentum must be equivalent to the reciprocal of the length of a wave whose normal coincides with the direction of momentum. The wave in question must not be imagined as existing in ordinary three-dimensional space, but in so-called configuration space, the dimension of which is given by the number of degrees of freedom of the system, and in which the square of the element of length is measured by twice the kinetic energy; or what comes to the same thing, by the square of the total momentum. It thus appears that the wave-length follows from the kinetic energy, that is from the difference between the constant total energy and the potential energy; this difference must be regarded as a function of position given beforehand.

The product of the frequency and the wave-length gives us the rate of propagation of the wave; in other words, it gives us the phase-velocity of a given wave—the so-called material

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[1] A vector is a quantity which has a definite direction; for example, "100 miles per hour East" (or any other direction) is a vector. [TRANS.]

wave—in configuration space. If the appropriate values are substituted in the familiar equation of classical mechanics, we obtain the linear homogeneous partial differential equation set up by Schrödinger. This equation has provided the basis of modern Quantum-mechanics, in which it seems to play the same part as do the equations established by Newton, Lagrange and Hamilton in classical mechanics. Nevertheless there is an important distinction between these equations, consisting in the fact that in the latter equations the co-ordinates of the configuration point are not functions of time, but independent variables. Accordingly, while for any given system the classical equations of motion were more or less numerous and corresponded to the number of degrees of freedom of the system, there can be only one single quantum-equation for each system. In course of time the configuration point of classical theory describes a definite curve; on the other hand, the configuration point of the material wave fills at any given time the whole of infinite space, including those parts of space where potential energy is greater than the total energy, so that according to the classical theory, kinetic energy would become negative in these parts of space, and the momentum imaginary.

This case resembles the so-called total reflection of light, where according to geometrical optics light is completely reflected, because the angle of refraction becomes imaginary; whereas according to the wave-theory of light, it is perfectly possible for light to penetrate into the second medium, even if it cannot do so as a plane wave.

At the same time, the fact that there are points in configuration space where the potential energy exceeds the total energy is of extreme importance for Quantum-mechanics. Calculation shows that in every such instance a finite wave corresponds not to any given value of the energy constant, but corresponds only to certain definite values:—the so-called characteristic energy-values, which can be calculated from the wave-equation and have different values according to the nature of the given potential energy.

From the discrete characteristic energy-values, discrete characteristic values of the period of oscillation may be derived. The latter are determined according to the Quantum postulate, in a similar manner to that of a stretched cord with fixed ends; with this distinction that the latter quantization is determined by an external condition, viz. the length of the cord, whereas in the present instance it depends upon the

Quantum of Action, which in turn depends directly upon the differential equation.

To each characteristic vibration there corresponds a particular wave-function (ψ); this is the solution of the wave-equation; and all these different characteristic functions form the component elements for the description of any movement in terms of wave-mechanics.

Thus we reach the following result: in classical Physics the physical system under consideration is divided spatially into a number of smallest parts; by this means the motion of material bodies is traced back to the motion of their component particles, the latter being assumed to be unchangeable. In other words, the explanation is based upon a theory of corpuscles. Quantum Physics, on the other hand, analyses all motion into individual and periodic material waves, which are taken to correspond to the characteristic vibrations and characteristic functions of the system in question; in this way it is based upon wave-mechanics. Accordingly, in classical mechanics the simplest motion is that of an individual particle, whereas in quantum-mechanics the simplest motion is that of a simple periodic wave; according to the first, the entire motion of a body is taken as being the totality of the motions of its component particles; whereas according to the second, it consists in the joint effect of all kinds of periodic material waves. To illustrate the difference between these two views, we may once more refer to the vibrations of a stretched cord. On the one hand these vibrations may be imagined as consisting of the sum of the motions of the different particles of the cord, where each particle is in motion independently of all the rest and in accordance with the force acting upon it, which in turn depends upon the local curvature of the cord. On the other hand the process of vibration may be analysed into the fundamental and upper partial vibrations of the cord, where each vibration affects the cord in its totality and the sum total of vibration is the most general kind of motion taking place in the cord.

Wave-mechanics also furnishes an explanation for another fact which hitherto has been inexplicable. According to Niels Bohr's theory, the electrons of an atom move around the nucleus in accordance with laws very similar to those which govern the motion of the planets around the sun. Here the place of gravitation is taken by the attraction between the opposite charges of the nucleus and the electrons. There is, however, a curious distinction, consisting in the fact that the electrons can move only in definite orbits distinct from each other, whereas with the planets no one orbit appears to be privileged beyond any other. According to the wave theory of electrons this circumstance, at first sight unintelligible, is easily explained. If the orbit of an electron returns upon itself, it is clear that it must comprise an integral number of wavelengths, just as the length of a chain which forms a complete circle, if it consists of a number of equal links, must always equal an integral number of link-lengths. According to this view the revolution of an electron around the nucleus is not so much like the movement of a planet around the sun, as like the rotation of a symmetrical ring upon its centre, so that the ring as a whole retains the same position in space; thus there is no physical meaning in referring to the local position of the electron at any instant.

The following question may now be asked: if motion is to be analysed not into particles, but into material waves, what is the procedure of wave-mechanics when it is called upon to describe the motion of a single particle which occupies a given position at a given time? The answer to this question throws light upon the great contrast between the two theories with which we have been dealing. In the first instance we must examine the physical meaning of the wave function ψ of a simple periodic material wave. This meaning can be derived from the consideration that the energy of a material wave has a twofold meaning. It is true that it denotes the period of vibration of the wave; but of course it does not follow from this that it has lost its original meaning, which it derives from the principle of conservation of energy. But if the energy principle is to apply to wave-mechanics, then it must be possible to represent the energy of a material wave, not only by the frequency of its vibrations, but also by means of an integral comprehending the entire configuration space of the wave.

In fact, then, if the wave-equation is multiplied by $\bar{\psi}$ and the product is integrated over the entire configuration space, there results a definite expression for the energy, which can be most vividly interpreted in the following manner.

We imagine the material system of particles under consideration to be multiplied many times, and we further imagine that each of the resulting systems is in a different configuration, so that we obtain a very great number of particles in configuration space. We further allot to the configuration points existing in the different infinitely small elements of space a definite energy which is composed (*a*) of the value of the local potential energy (which is given beforehand) and (*b*) of a second element which varies as the square of the local gradient of ψ, and which we can interpret as being equivalent to kinetic energy. If, then, the spatial density of the configuration points at any one place is assumed to be equal to the square of the absolute value of ψ (which latter we may assume to have any magnitude we desire, since one of the constant factors of ψ can be selected by ourselves at will), it follows that the mean energy of all the configuration points is equivalent to the energy of the material wave. Accordingly the absolute value of the amplitude of the wave has no meaning whatever in a physical sense. If we imagine ψ to be selected in such a way that the square of the absolute value of ψ, when integrated over the configuration space, gives us the value 1, then we can also say that this square denotes the probability that the material system of particles is actually existing at the point in question within the configuration space. Thus we have found a vivid expression for the physical meaning of the wave-function ψ, which we were looking for.

In the course of all these considerations we had assumed that ψ had a definite characteristic function of its own, and that there was a simple periodic wave corresponding to it. Similar statements, however, may be made for the general case where waves having different periods are superimposed. In that case the wave-function ψ is the algebraic sum of the periodic characteristic functions multiplied by a certain amplitude constant, and once again the square of the absolute value of ψ denotes the probability for the corresponding position of the configuration point. In the general case, of course, we can no longer speak of one single definite period of vibration of the material waves; on the other hand, however, we can still speak of a definite energy. Accordingly the Quantum-equation $E = h\nu$ loses its original meaning and only gives us an average frequency ν. It is worth noting here that if a sufficiently large number of different waves having approximately equal frequencies are superimposed, the wave-function of the resulting wave is the sum of the individual wave-functions; its energy on the other hand does not increase proportionately with the number of individual waves, but always retains its original mean value; the energy of a group of individual waves defines a mean frequency, and similarly the momentum of this group serves to define a mean wave-length.

To begin with, the amplitudes and phases of the individual waves can be selected at will. Beyond this, however, it is impossible to introduce further variety into the mechanical processes of which wave-mechanics can provide instances. This fact becomes important when we turn to the question raised above, in which we ask how the motion of a single definite particle is to be described in terms of wave-mechanics. It appears immediately that *such a description cannot be made in any exact sense.* Wave-mechanics possesses only one means of defining the position of a particle, or more generally the position of a definite point in configuration space; this consists in superimposing a group of individual waves of the system, in such a manner that their wave-functions cancel each other by interference everywhere within configuration space, and intensify each other only at the one point in question. In this case the probability of all the other configuration points would be equal to O, and would be equal to 1 only for the one point in question. In order to isolate this point completely we should, however, require infinitely small wavelengths, and consequently infinitely great momentum. Therefore, in order to obtain a result which would be even approximately useful, we should have to begin by substituting for the definite configuration point a finite (though still small) region of configuration space, or so-called wavegroup; this sufficiently expresses the fact that ascertaining the position of a configuration point is always in the wave theory affected by some sort of uncertainty.

If we wish to go further and ascribe to the system of particles a definite quantity of momentum as well as a definite configuration, then the Quantum postulate, if taken strictly, will allow us to make use of only one single wave of a definite length for our exposition, and once more description becomes impossible. On the other hand, if a slight uncertainty is allowed to creep into the quantity of momentum, then we can reach our goal, at least approximately, if we make use of the wave within a certain narrow range of frequency.

According to wave-mechanics, both the position and the momentum of a system of particles can never be defined without some uncertainty. Now the fact is that between these two kinds of uncertainty there is a definite relation. This follows from the simple reflection that if the waves of which we make use are to cancel each other through interference outside the above-mentioned small configuration region, then in spite of their small difference in frequency, noticeable differences in propagation must appear at the opposite boundaries of the region. If in accordance with the Quantum postulate, we substitute differences of momentum for differences of propagation, we obtain Heisenberg's Principle, which states that the product of the uncertainty of position and uncertainty of momentum is at least of the same order of magnitude as the quantum of action.]

The more accurately the position of the configuration point is ascertained, the less accurate is the amount of momentum; and conversely. These two kinds of uncertainty are thus in a certain sense complementary; this complementariness is limited by the fact that momentum can under certain conditions be defined with absolute accuracy in wave-mechanics, whereas the position of a configuration point always remains uncertain within a finite region.

Now this relation of uncertainty, established by Heisenberg, is something quite unheard of in classical mechanics. It had always been known, of course, that every measurement is subject to a certain amount of inaccuracy; but it had always been assumed that an improvement in method would lead to an improvement in accuracy, and that this process could be carried on indefinitely. According to Heisenberg, however, there is a definite limit to the accuracy obtainable. What is most curious is that this limit does not affect position and velocity separately, but only the two when combined together. In principle, either taken by itself can be measured with absolute accuracy, but only at the cost of the accuracy of the other.

Strange as this assertion may seem, it is definitely established by a variety of facts. I will give one example to illustrate this. The most direct and accurate means of ascertaining the position of a particle consists in the optical method, when the particle is looked at with the naked eye or through a microscope, or else is photographed. Now for this purpose the particle in question must be illuminated. If this is done the definition becomes more accurate; consequently the measurement becomes more exact in proportion as the light-waves employed become shorter and shorter. In this sense, then, any desired degree of accuracy can be attained. On the other hand there is also a disadvantage, which affects the measurement of velocity. Where the masses in question have a certain magnitude, the effect of light upon the illuminated object may be disregarded. But the case is altered if a very small mass, e.g. a single electron, is selected; because each ray of light, which strikes the electron and is reflected by it, gives it a distinct impulse; and the shorter the light-wave the more powerful is this impulse. Consequently, the shorter the light-wave the more accurately is it possible to determine position; but at the same moment measurement of velocity becomes proportionately inaccurate; and similarly in analogous instances.

On the view which has just been set out classical mechanics, which is based on the assumption of unchanging and accurately measurable corpuscles moving with a definite velocity, forms one ideal limiting case. This ideal case is actually realized when the observed system possesses a relatively considerable energy. When this happens, the distinct characteristic energy values will lie close to each other, and a relatively small region of energy will contain a considerable number of high wave-frequencies (i.e. of short wave-lengths); through the superposition of these a small wave-group with definite momentum can be delimited comparatively accurately within the configuration space. In this case, wave-mechanics merges with the mechanics of particles; Schrödinger's differential equation becomes the classical differential equation of Hamilton and Jacobi, and the wave-group travels in configuration space in accordance with the same laws which govern the motion of a system of particles according to classical mechanics. But this state of affairs is of a limited duration; for the individual material waves are not interfering continually in the same manner, and consequently the wave-group will disintegrate more or less quickly; the position of the relative configuration point will become more and more uncertain, and finally the only quantity remaining that is accurately defined is the wave-function ψ.

The question now arises whether these conclusions correspond with experience. Since the Quantum of Action is so small, this question can be answered only within the framework of atomic physics; consequently the methods employed will always be extremely delicate. At present we can only say that hitherto no fact has been discovered which throws doubt on the applicability in Physics of all these conclusions.

The fact is that since the wave-equation was first formulated, the theory has been developing at a most remarkable rate. It is impossible within the framework of a small volume to mention all the extensions and applications of the theory which have been evolved within recent years. I shall confine myself to the so-called stress of protons and electrons; the formulation of Quantum-mechanics in terms of Relativity; the application of the theory to molecular problems, and the treatment of the so-called "many-body problem", i.e. its application to a system containing a number of exactly similar particles. Here statistical questions, relating to the number of possible states within a system, having a given energy, are particularly important; they also have a bearing on the calculation of the entropy of the system.

Finally, I cannot here enter in detail upon the Physics of light-quanta. In a certain sense this study has developed in the opposite direction from the Physics of particles. Originally Maxwell's theory of electromagnetic waves dominated this region, and it was not seen until later that we must assume the existence of discrete light-particles; in other words that the electromagnetic waves, like the material waves, must be interpreted as waves of probability.

Perhaps there is no more impressive proof of the fact that a pure wave theory cannot satisfy the demands of modern Physics any more than a pure corpuscular theory. Both theories, in fact, represent extreme limiting cases. The corpuscular theory, which is the basis of classical mechanics, does justice to the configuration of a system, but fails to determine the values of its energy and of momentum; conversely the wave theory, which is characteristic of classical electrodynamics, can give an account of energy and momentum, but excludes the idea of the localization of light-particles. The standard case is represented by the intermediate region, where both theories play equally important parts; this region can be approached from either side, although at present a close approach is impossible. Here many obscure points await solution, and it remains to be seen which of the various methods employed for their solution best leads to the goal. Among them we may mention the matrix calculus invented by Heisenberg, Born, and Jordan, the wave theory due to de Broglie and Schrödinger, and the mathematics of the q numbers introduced by Dirac.

§ 4

If we attempt to draw a comprehensive conclusion from the above description and to obtain an insight into the distinguishing characteristics of our new picture of the world, the first impression will no doubt be somewhat unsatisfactory. First of all it will appear surprising that wave-mechanics, which itself is in complete contradiction to classical mechanics, nevertheless makes use of concepts drawn from the classical corpuscular theory; e.g. the concept of the co-ordinates and momentum of a particle, and of the kinetic and potential energy of a system of particles. The contradiction is the more surprising since it afterwards proved impossible simultaneously to determine exactly the position and momentum of a particle. At the same time these concepts are absolutely essential to wave-mechanics; for without them it would be impossible to define configuration space and ascertain its measurements.

There is another difficulty attached to the wave theory, consisting in the fact that material waves are not as easy to bring before the imagination as are acoustic or electromagnetic waves; for they exist in configuration space instead of ordinary space, and their period of vibration depends on the choice of the physical system to which they belong. The more extensive this system is assumed to be, the greater will be its energy, and with this the frequency.

It must be admitted that these are serious difficulties. It will be possible, however, to overcome them if two conditions are fulfilled:—the new theory must be free from internal contradictions; and its applied results must be definite and of some significance for measurement. At the present time opinions are somewhat divided whether these requirements are fulfilled by Quantum-mechanics, and if so, to what extent. For this reason I propose to discuss this fundamental point further.

It has frequently been pointed out that Quantum-mechanics confines itself on principle to magnitudes and quantities which can be observed, and to questions which have a meaning within the sphere of Physics. This observation is correct; but in itself it must not be considered a special advantage of the Quantum Theory as opposed to other theories. For the question whether a physical magnitude can in principle be observed, or whether a certain question has a meaning as applied to Physics, can never be answered *a priori*, but only from the standpoint of a given theory. The distinction between the different theories consists precisely in the fact that according to one theory a certain magnitude can in principle be observed, and a certain question have a meaning as applied to Physics; while according to the other theory this is not the case. For example, according to the theories of Fresnel and Lorentz, with their assumption of a stationary ether, the absolute velocity of the earth can in principle be observed; but according to the Theory of Relativity it cannot; again, the absolute acceleration of a body can be in principle observed according to Newtonian mechanics, but according to Relativity mechanics it cannot. Similarly the problem of the construction of a *perpetuum mobile* had a meaning before the principle of the conservation of energy was introduced, but ceased to have a meaning after its introduction. The choice between these two opposed theories depends not upon the nature of the theories in themselves, but upon experience. Hence it is not sufficient to describe the superiority of Quantum-mechanics, as opposed to classical mechanics, by saying that it confines itself to quantities and magnitudes which can in principle be observed, for in its own way this is true also of classical mechanics. We must indicate the particular magnitudes or quantities which, according to Quantum-mechanics, are or are not in principle observed; after this has been done it remains to demonstrate that experience agrees with the assertion.

Now this demonstration has in fact been completed, e.g. with respect to Heisenberg's Principle of Uncertainty, so far as seems possible at the present moment, and to this extent it can be looked upon as proving the superiority of wave-mechanics.

In spite of these considerable successes, the Principle of Uncertainty which is characteristic of Quantum Physics has caused considerable hesitation, because the definition of magnitudes and quantities which are continually in use is in principle treated as being inexact by this theory. This dissatisfaction is increased by the fact that the concept of probability has been introduced in the interpretation of the equations used in Quantum-mechanics; for this seems to imply a surrender of the demands of strict causality in favour of a form of indeterminism. To-day, indeed, there are eminent physicists who under the compulsion of facts are inclined to sacrifice the principle of strict causality in the physical view of the world

If such a step should actually prove necessary the goal of physicists would become more remote; and this would be a disadvantage whose importance it is impossible to overestimate. For in my opinion, so long as any choice remains, determinism is in all circumstances preferable to indeterminism, simply because a definite answer to a question is always preferable to an indefinite one.

So far as I can see, however, there is no ground for such a renunciation. For there always remains the possibility that the reason why it is impossible to give a definite answer resides, not in the nature of the theory, but in the manner in which the question is asked. If a question is inadequately formulated physically, the most perfect physical theory can give no definite answer; a fact widely known in classical statistics and frequently discussed. For example, if two elastic spheres strike one another in a plane, while their velocities before impact and the laws of impact are known in all their details, it still remains impossible to state their velocities after impact. The fact is that, in order to calculate the four unknown components of the velocities of the two spheres after impact, we have only three equations derived from the conservation of energy and the two components of momentum. From this, however, we do not infer that there is no causality governing impact phenomena; what we do say is that certain essential data are missing which are requisite for their complete determination.

In order to apply these considerations to the problems of Quantum Physics, we must now return to the arguments dealt with in the Introduction.

If it is really true that, in its perpetual changes, the structure of the physical world-view moves further and further away from the world of the senses, and correspondingly approaches the real world (which, as we saw, cannot in principle be apprehended at all), then it plainly follows that our view of the world must be purged progressively of all anthropomorphic elements. Consequently we have no right to admit into the physical world-view any concepts based in any way upon human mensuration. In fact this is not the case with Heisenberg's Principle of Uncertainty: this was reached from the consideration that the elements of the new view of the world are not material corpuscles, but simple periodic material waves which correspond to the physical system under consideration—a conclusion obtained in accordance with the mathematical principle that it is impossible to determine a definite particle with definite momentum by means of superposition of simple periodic waves having a finite length. The principle has nothing whatever to do with any measurement, while the material waves are definitely determined by means of the mathematical problem of boundary values relating to the case in question. Here there is no question of indeterminism.

The question of the relation between the material waves and the world of sense is a different one. For this relation renders it possible for us to become acquainted with physical events; if a system were completely cut off from its surroundings we could never know of its existence.

At first glance it appears that this question has nothing to do with Physics, since it belongs partly to Physiology and partly to Psychology. These objections, however, lead to no real difficulty. It is always possible to imagine suitably constructed instruments being substituted for human senses, e.g. self-registering apparatus like a sensitive film, which registers the impressions derived from the environment, and is thus capable of furnishing evidence about the events taking place in these surroundings. If such instruments are included within the physical system which we propose to consider, and if all other influences are eliminated, then we have a physical system cut off from the rest of the world of which we can discover something by means of measurement; although it is true that we must take into account the structure of the measuring instruments, and the reaction which they might conceivably have upon the events which we desire to measure.

If we possessed an instrument reacting to a simple periodic material wave in the same way as a resonator reacts to a sound-wave, then we would be in a position to measure individual material waves and thus to analyse their behaviour. This is not the case; the fact is that the indications given by such instruments as we possess, e.g. the darkening of a photographic film, do not allow us to make a safe inference about all the details of the process under examination. We have no right, however, to infer from this that the laws of material waves are indeterminate.

Another and more direct attempt might be made to substantiate the assumption of indeterminism from the fact that, according to wave-mechanics, the events within a system of particles cut off from the outside world are not determined in any way by the initial state of the system, i.e. by the initial configuration and initial momentum. There is not even an approximate determination; for the wave-group corresponding to the initial state will in time disintegrate generally and fall apart into individual waves of probability.

On closer consideration, however, we see that in this instance the element of indeterminism is due to the manner in which the question is asked. The question is based upon corpuscular mechanics; and in corpuscular mechanics the initial state governs the course of the event for all time. But in wave-mechanics such a question has no place, if only because the final result is on principle affected with a finite inaccuracy due to the Principle of Uncertainty.

Since the times of Leibniz, on the other hand, another form of question in classical mechanics has been known which in this sphere leads to a definite answer. An event is completely determined for all time if, apart from the configuration at a certain time, we know, not the momentum, but the configuration of the same system at a different instant. In this case the principle of variation, or principle of least action, is used in order to calculate the event. To take the previous example, where two elastic spheres meet in a plane, if we know the initial and final position of the spheres and the interval between those two positions, then the three unknown quantities, namely the two local co-ordinates and the time co-ordinate of impact, are completely determined by the three equations of conservation.

This changed formulation of the problem differs from the previous formulation because it is immediately applicable to wave-mechanics. It is true, as we saw, that a given configuration can never be defined with complete accuracy by the wave theory; but on the other hand it is theoretically possible to reduce the uncertainty below any desired limit, and thus to determine the event in question with any desired degree of accuracy. Further, the disintegration of wave-groups is no evidence in favour of indeterminism, since it is equally possible for a wave-group to conglomerate: in both the wave theory and the corpuscular theory the *direction* of the process is immaterial. Any movement might equally well take place in the opposite direction.

When the above formulation of the problem is adopted a given wave-group generally, of course, exists only at the two selected instants: in the intervening period, as well as before and after the process, the different elementary waves will exist separately. But whether they are described as material waves or as waves of probability, in either case they will be completely determined. This is the explanation of the apparent paradox, that when a physical system passes by a definite process from one definite configuration during a definite time into some other definite configuration, the question what its configurations are during the intervening period has no physical significance; similarly on this view there is no meaning in the question of what is the track of light quantum emitted from a point source and absorbed at a given point on an observation-screen.

It should at the same time be emphasized that on this view the meaning of determinism is not exactly what it is in classical Physics. In the latter the configuration is determined; in Quantum Physics, the material waves. The distinction is important, because the connection between the configuration and the world of sense is far more direct than that between the material waves and the sense-world. To this extent the relation between the physical world-view and the world of sense appears to be considerably looser in modern Physics.

This is undoubtedly a disadvantage; but it is the price that must be paid in order to preserve the determinism of our world-view. And further, this step appears to lie in the general direction in which Physics is actually developing; this has been pointed out on more than one occasion, since in the course of its progressive evolution, the structure of the physical view of the world is moving farther and farther away from the world of sense, and assuming more and more abstract forms. Indeed, the principle of Relativity seems actually to demand such a view; for on this principle Time stands on the same level with Space, whence it follows that, if a finite space is required for the causal description of a physical process, a finite temporal interval must also be used in order to complete the description.

On the other hand, it may well be that the suggested formulation of the question is too one-sided, and too anthropomorphic to furnish satisfactory material for a new theory of the structure of the physical world; it may be that we shall have to look for some other formulation. In any case many complex problems remain to be solved, and many obscure points to be cleared up.

In view of the peculiar difficulties of the position which has been reached by theoretical Physics, a feeling of doubt persists whether the theory, with all its radical innovations, is really on the right path. The answer to this decisive question depends wholly upon the degree of necessary contact with the sense world which the physical world-view maintains in the course of its incessant advance. If this contact is lost even the most perfect world-view would be no better than a bubble ready to burst at the first puff of wind. There is, fortunately, no cause for apprehension, at least in this respect: indeed we may assert without exaggeration that there was no period in the history of Physics when theory and experience were linked so closely together as they are now. Conversely, it was the facts learned from experiments that shook and finally overthrew the classical theory. Each new idea and each new step were suggested to investigators, where it was not actually thrust upon them, as the result of measurements. The Theory of Relativity was led up to by Michelson's experiments on optical interference, and the Quantum Theory by Lummer's, Pringsheim's, Ruben's and Kurlbaum's measurements of the spectral distribution of energy, by Lenard's experiments on the photoelectric effect, and by Franck and

Hertz's experiments on the impact of electrons. It would lead me too far if I were to enter on the numerous and surprising results which have compelled Physical theory to abandon the classical standpoint and to enter on a definite new course.

We can only hope that no change will take place in this peaceful international collaboration. It is in this reciprocal action of experiment and theory—which is at once a stimulus to and a check upon progress—that we see the surest and indeed the only guarantee of the future advance of Physics.

What will be the ultimate goal? I had occasion at the beginning to point out that research in general has a twofold aim—the effective domination of the world of sense, and the complete understanding of the real world; and that both these aims are in principle unattainable. But it would be a mistake to be discouraged on this account. Both our theoretical and practical tangible results are too great to warrant discouragement; and every day adds to them. Indeed, there is perhaps some justification for seeing in the very fact that this goal is unattainable, and the struggle unending, a blessing for the human mind in its search after knowledge. For it is in this way that its two noblest impulses—enthusiasm and reverence—are preserved and inspired anew.

§ 5

What now do we mean by physical law? A physical law is any proposition enunciating a fixed and absolutely valid connection between measurable physical quantities—a connection which permits us to calculate one of these quantities if the others have been discovered by measurement. The highest and most keenly desired aim of any physicist is to obtain the most perfect possible knowledge of the laws of Physics, whether he looks at them from a utilitarian point of view and values them because they enable him to save himself the trouble of costly measurements, or takes a deeper view and looks to them for satisfaction of a profound yearning after knowledge and for a firm basis of natural science.

How do we discover the individual laws of Physics, and what is their nature? It should be remarked, to begin with, that we have no right to assume that any physical laws exist, or if they have existed up to now, that they will continue to exist in a similar manner in future. It is perfectly conceivable that one fine day Nature should cause an unexpected event to occur which would baffle us all; and if this were to happen we would be powerless to make any objection, even if the result would be that, in spite of our endeavours, we should fail to introduce order into the resulting confusion. In such an event, the only course open to science would be to declare itself bankrupt. For this reason, science is compelled to begin by the general assumption that a general rule of law dominates throughout Nature, or, in Kantian terminology, to treat the concept of causality as being one of the categories which are given *a priori* and without which no kind of knowledge can be attained.

From this it follows that the nature of the laws of Physics, and the content of these laws, cannot be obtained by pure thought; the only possible method is to turn to the investigation of Nature, to collect the greatest possible mass of varied experiences, to compare these and to generalize them in the simplest and most comprehensive proposition. In other words, we must have recourse to the method of induction.

The content of an experience is proportionally richer as the measurements upon which it is based are more exact. Hence it is obvious that the advance of physical knowledge is closely bound up with the accuracy of physical instruments and with the technique of measurement. The latest developments of Physics provide us with striking examples of the truth of this. Measurement alone, however, does not suffice. For each measurement is an individual event standing by itself; as such, it is determined by special circumstances, especially by a definite place and a definite time, but also by a definite measuring instrument, and by a definite observer. It is true that frequently the generalization which is our object is quite obvious and, so to speak, thrusts itself upon us; on the other hand, there are cases where it is extremely difficult to find the common law governing a number of different measurements, either because it seems impossible to find such a law, or because a number of different laws seem available in order to generalize the facts. Both possibilities are equally unsatisfactory.

In such cases, the only method of advance consists in introducing a so-called working hypothesis to see what it is worth and how far it will lead. It is generally a sign that the hypothesis is likely to turn out useful if it works even in those regions for which it was not originally designed. In such a case we have a right to assume that the law which it enunciates has a deeper meaning and opens the way to unmistakably new knowledge.

We see then that a good working hypothesis is essential for inductive investigation. This being so, we are faced with the difficult question how we are to set about to find the most suitable hypothesis. For this there can be no general rule. Logical thought by itself does not suffice—not even where it has an exceptionally large and manifold body of experience to aid it. The only possible method consists in immediately gripping the problem or in seizing upon some happy idea. Such an intellectual leap can be executed only by a lively and independent imagination and by a strong creative power, guided by an exact knowledge of the given facts so that it follows the right path.

Such an intellectual process generally consists in the introduction of certain mental images and analogies which point the way to the reigning laws already known in other regions, thus suggesting a further step towards the simplification of the physical view of the world.

It is precisely at these points where success seems to be awaiting us, however, that a serious danger is frequently hidden. Once a step forward has succeeded and the working hypothesis has demonstrated its usefulness, we must go further. We have to reach the actual essence of the hypothesis and, by suitably formulating it, we have to throw light upon its genuine content by eliminating everything that is inessential. This process is not as simple as it might appear. The intellectual leap of which we spoke above constructs a kind of bridge by which we can approach fresh knowledge; but on closer examination it frequently appears that this bridge is merely provisional, and that a more enduring structure must be put in its place, capable of bearing the heavy artillery of critical logic.

We must bear in mind that every hypothesis is the outcome of the efforts of imagination, and that imagination works through direct intuition. But in Physics, as soon as we come to look for a rational theory or a logical demonstration, direct intuition is a very doubtful ally, however indispensable it may be while we are forming our hypothesis. For while it is natural that we should rely upon imaginations and ideas of this kind, which proved fruitful in one direction or another, such reliance is only too apt to lead to an overestimation of their importance and to untenable generalizations. We must further recognize that the authors of a new and practicable theory are frequently little inclined to introduce any important changes in the groups of ideas which led them towards their discoveries, whether from indolence or from a certain sentimental feeling, and that they often exert the whole of their well-earned authority in order to be able to maintain their original standpoint. Thus we shall readily understand the difficulties which often stand in the way of healthy theoretical development. Examples may be found at every point in the history of Physics, and I propose to enumerate some of the more important of them.

The first exact measurements were made in the region of Space and Time—the first region where accurate measurement was possible. Hence naturally the earliest physical laws were discovered in this field; in other words, in the sphere of mechanics. Again, we can readily understand how it came about that the first laws which were established related to those motions which occur regularly and independently of external interference, namely, the motions of the celestial bodies. We know that the civilized peoples of the

East had discovered thousands of years ago how to derive from their observations formulæ which allowed them to calculate in advance the motion of the sun and the planets with great accuracy. Each improvement in the instruments of measurement was accompanied by an improvement of the formulæ. By their co-ordination and comparison the theories of Ptolemy, Copernicus and Kepler were evolved in course of time, each of which is simpler and more exact than those which preceded it. All these theories are alike in endeavouring to answer the question, what is the connection between the position of a celestial body, a planet for example, and the moment of time at which it occupies this position? The nature of this necessary connection is, of course, different for the different planets, and this in spite of the fact that the motions of the planets have many characteristics in common.

The decisive step beyond this type of question was taken by Newton. Newton summed up all the formulæ relating to the planets in one single law governing their motion, and indeed that of all the celestial bodies. He was enabled to do this because he made the law of motion independent of the particular moment to which it is applied: for the instant he substituted the time-differential. Newton's theory of planetary motion enunciates a fixed connection not between the position of a planet and time, but between the acceleration of a planet and its distance from the sun. Now this law—a vectorial differential equation—is the same for all the planets. Hence if the position and velocity of a planet are known for any moment, then its motion for all time can be exactly calculated.

The successes obtained as the result of the further application of Newton's formulation of the laws of motion prove that it is not merely a new description of certain natural phenomena, but that it represents a real advance in the understanding of actual facts. It is not merely more exact than Kepler's formulæ, for example when it allows for the interference in the elliptical orbit of the earth around the sun due to the periodic proximity of Jupiter, where formula and measurement are in exact agreement; more than that, it also covers the motion of such bodies as comets, twin stars, etc., which altogether elude Kepler's laws. The complete and immediate success of Newton's theory was due, however, to the fact that when applied to motion occurring on the earth, it led to the same numerical laws of gravitation and pendulum movements which

Galileo had already discovered by measurement, and also threw light on otherwise inexplicable phenomena, such as those of tides, rotation of the plane of the pendulum, precession of the axis of rotation, etc.

The question which especially interests us at the moment is how Newton reached his differential equation for planetary motion. He did not reach it by establishing a connection between the acceleration of a planet and its distance from the sun, and by looking for a numerical connection between them; what he did was first to forge an intellectual link between them, leading from the concept of the position of a planet to that of its acceleration; and this link he called Force. He assumed that the position of a planet relatively to the sun depends upon a force of attraction directed towards the sun, and that the same attractive force also causes a definite change in the planet's motion. This was the germ of the law of gravitation, as well as of the law of inertia. The notion of force was no doubt derived (as the word implies) from the idea of the muscular sensation which arises when a weight is lifted or a ball is thrown; this idea was generalized and applied to every kind of change of motion, even where the forces in question are so great that no human power could possibly suffice to effect them.

Small wonder, then, that Newton attributed the greatest importance to the concept of force which had helped him to reach such striking results. At the same time it must be noted that this concept does not occur in the law of motion proper. Newton looked to the concept of force for an explanation of every change of motion; and thus it came about that Newtonian force was regarded as the main and fundamental concept in mechanics, and not only in mechanics, but also in Physics; so that, in course of time, physicists formed the habit of making their first question when dealing with physical phenomena: what force is here in action?

Recent developments in Physics present a certain contrast with Newton's theory, so that in a manner it is true to say that the concept of force is no longer of fundamental importance for physical theory. In modern mechanics force is no more than a magnitude of secondary importance, and its place has been taken by higher and more comprehensive concepts—that of work[1] or

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[1] "Work" is here used in its scientific sense of the product of the force and the distance through which the force acts. [TRANS.]

potential, where force in general is defined as a negative potential gradient.

It might here be objected that work surely cannot be looked upon as something primary, since there must be some kind of force in existence that does the work. This kind of argument is of the physiological and not of the physical order. It is true that in lifting a weight the contraction of the muscles and the accompanying sensations are primary, and are the cause of the motion which actually takes place. But this kind of work, which is a physiological process, must be clearly distinguished from the physical force of attraction with which alone we are here concerned; it is this force which the earth exerts upon everything having weight; and this in its turn depends upon the gravitational potential which is already in existence and is primary.

The idea of potential is superior to that of force, partly because it simplifies the laws of Physics, and also because the significance of the idea of potential has a far greater scope than that of force; it reaches beyond the sphere of mechanics into that of chemical affinities, where we are no longer concerned with Newtonian force. It must be admitted that the idea of potential has not the advantage of immediate obviousness which belongs to force by virtue of its anthropomorphic quality; whence it follows that the elimination of the concept of force renders the laws of Physics much less obvious and easy of understanding. Yet this development is quite natural; the laws of Physics have no consideration for the human senses; they depend upon facts, and not upon the obviousness of facts.

In my opinion, the teaching of mechanics will still have to begin with Newtonian force, just as optics begins with the sensation of colour, and thermodynamics with the sensation of warmth, despite the fact that a more precise basis is substituted later on. Again, it must not be forgotten that the significance of all physical concepts and propositions ultimately does depend on their relation to the human senses. This is indeed characteristic of the peculiar methods employed in physical research. If we wish to form concepts and hypotheses applicable to Physics, we must begin by having recourse to our powers of imagination; and these depend upon our specific sensations, which are the only source of all our ideas. But to obtain physical laws we must abstract exhaustively from the images introduced, and remove from the definitions set up all irrelevant elements and all imagery which do not stand in a logical connection with the measurements obtained. Once we have formulated physical laws, and reached definite conclusions by mathematical processes, the results which we have obtained must be translated back into the language of the world of our senses if they are to be of any use to us. In a manner this method is circular; but it is essential, for the simplicity and universality of the laws of Physics are revealed only after all anthropomorphic additions have been eliminated.

The concept of Force as used by Newton is only one of a number of intellectual links and auxiliary notions employed in order to render an idea more intelligible. In this connection I should like to mention the idea of osmotic pressure introduced by van't Hoff. This idea proved particularly fruitful in physical chemistry, where it was used in order to formulate the physical laws of solutions, especially of the freezing-point and steam-pressure. To obtain instances of osmotic pressure, and measure it accurately, is not altogether easy, since an extremely complex apparatus (the so-called semi-permeable membranes) is required. We must the more admire the intuitive insight which led van't Hoff to formulate the laws known under his name despite the scantiness of the observed facts at his disposal. Yet in their present form these laws require osmotic pressure no more than the laws of motion require Newtonian Force.

Besides the above there are other kinds of intellectual aids which assist imagination, and have proved of great assistance in the formation of working hypotheses, but which in the further course of development actually embarrassed later progress. One of these is particularly worth mention here. Men had accustomed themselves to see in some kind of force the cause underlying every natural change; and thus they were all the more disposed to imagine every invariable and constant magnitude or quantity as being of the nature of a *Substance*. From the earliest times the concept of Substance has played an important part in Physics: but closer examination shows that this has not always been helpful. It is, of course, easy to see that wherever conservation is concerned, it is possible to assume a Substance of which conservation is predicated; and such an assumption undoubtedly makes it easier to grasp the meaning of the principle, and hence facilitates its use. A magnitude which in spite of every change retains its quantity surely cannot be imagined more vividly than in the shape of a moving material body. It is a feature of this tendency that we are so prone to interpret all natural events as being movements of masses of substance—a mechanistic interpretation. For example, the origin and distribution of light were explained by wave-motion in a substantial light ether; the chief laws of optics were described in this manner and found to agrec with experience —until the moment came when the mechanistic theory of Substance failed and became lost in unfruitful speculation.

Again, for a time, the concept of Substance proved exceedingly useful as applied to Heat. The careful development of calorimetry, during the first half of the last century, was due in the main to the assumption that an unchanging heat-substance flowed from the warmer into the colder body. When it was shown that in these circumstances the amount of heat can be increased (e.g. by friction) the Substance theory defended itself by appealing to supplementary hypotheses. But although this method helped for some time, it did not avail indefinitely.

In the theory of electricity the dangerous consequences of an exaggerated application of the idea of Substance became obvious at an early stage. Here again the idea of a subtle and quickmoving electrical substance, giving rise to certain manifestations of force, serves admirably in order to render plastic before the mind such principles as that of the invariability of the quantity of electricity, and such subsidiary ideas as those of the electrical current and of the reciprocal action of charged conductors carrying a current. Here again, however, the analogy fails as soon as we have to allow for the fact that this view implies the assumption of the existence of two opposite substances, one positive and one negative, which completely neutralize each other when they are combined. Such an occurrence is at least as unthinkable as the creation of two opposite substances (in the usual sense) out of nothing.

In this way we see that imaginative ideas and their resultant viewpoints must be used with the greatest caution, even when they have proved their value for some length of time, and despite the fact that they are indispensable for physical investigation and have provided the key to new knowledge on innumerable occasions. There is only one sure guide towards further development, and that is measurement, together with any logical conclusions that can be drawn from the concepts attached to this method. All other conclusions, and especially those characterized by their so-called self-evidence, should always be looked upon with a certain suspicion. The validity of a proof dealing with well-defined concepts is to be judged by reason and not by intuition.

§ 6

Up to this point we have been considering the manner in which the knowledge of physical laws is obtained. We will now proceed to examine the content and the essential nature of the laws of Physics in somewhat greater detail.

A physical law is generally expressed in a mathematical formula, which permits us to calculate the temporal succession of the events taking place in a certain physical system under certain definite and given conditions. From this point of view all the laws of Physics can be divided into two main groups.

The first group consists of those laws which remain valid even when the time order is reversed; in other words, when every process that fulfils their requirements can take place in the reversed order without running counter to them. The laws of mechanics and of electrodynamics are of this nature, except in so far as they relate to chemical phenomena and the phenomena of heat. Every purely mechanical or electrodynamic process can take place in the reverse direction. The movement of a body falling without friction is accelerated in accordance with the same law which governs the retardation of a body rising without friction; the same laws govern the movement of a pendulum to the left and to the right, and a wave can travel equally well in any direction and in any sense; a planet could equally well revolve around the sun westwards as eastwards. The question whether such movements could actually be reversed, and if so under what conditions, is another matter which need not here be discussed: we are now dealing with the law as such, not with the particular facts to which it applies.

The laws belonging to the second group are characterized by the fact that their time order is of essential importance, so that the events taking place in accordance with these laws have only one temporal direction and cannot be reversed. Among these processes we may mention all those in which heat and chemical affinity play a part. Friction is always accompanied by a decrease and never by an increase of relative velocity; where heat is conducted the warmer body always becomes cooler and the cooler body warmer; in diffusion the process invariably leads to a more thorough mixture and not to a progressive separation of the substances in question. Further, these irreversible events always lead to a definite final state; friction to a relative state of rest, the transfer of heat to temperature equilibrium, and diffusion to a completely homogeneous mixture. On the other hand the former class of reversible events knows neither beginning nor end, so long as no interference takes place from outside, but persists in incessant oscillation.

Now if we wish to introduce unity into the physical view of the universe we must somehow find a formula to cover both these contrasted types of law. How is this indispensable result to be brought about? Some thirty years ago theoretical physics was profoundly influenced by the so-called theory of Energetics, which sought to remove the antithesis by assuming that a fall in temperature, for example, was exactly analogous to the fall of a weight or of a pendulum from a higher to a lower position. This theory, however, did not take into consideration the essential fact that a weight can rise as well as fall, and that a pendulum has reached its greatest velocity at the moment when it has attained its lowest position and therefore, by virtue of its inertia, passes the position of equilibrium and moves to the other side. A transference of heat from a warmer to a colder body, on the contrary, diminishes with the diminution of the difference in temperature, while, of course, there is no such thing as any passing beyond the state of temperature equilibrium by reason of some kind of inertia.

In whatever way we look at it, the contrast between reversible and irreversible processes persists; it must therefore be our task to find some entirely new point of view which will allow us to see that after all there is some connection between the different types of laws. Perhaps we shall succeed in showing that one group of laws is a derivative of the other; if so, the question arises which is to be considered the more simple and elementary—the reversible processes or the irreversible.

Some light is thrown on this question by a formal consideration. Every physical formula contains a number of constant magnitudes, together with variable magnitudes which have to be determined by measurement from case to case. The former magnitudes are fixed once for all and give its characteristic form to the functional connection between the variables which is expressed in the formula. Now if we examine these constants more carefully, we shall find that they invariably are the same for the reversible processes, always recurring, however widely different are the attendant outer conditions. Among these are mass, the gravitation constant, the electrical charge and the velocity of light. On the other hand the constants of the irreversible processes, like the capacity for conducting heat, the coefficient of friction and the diffusion constant, depend to a greater or less degree on external circumstance, e.g. temperature, pressure, etc.

These facts naturally lead us to regard the constants of the first group as the simpler, and the laws dependent on them as the more elementary, and to suppose them incapable of further analysis, while treating the constants of the second group, and the laws depending on them, as being of a somewhat more complex nature. In order to test the validity of this assumption we must make our method of investigation somewhat more exact; we must, so to speak, apply a lens of greater power to the phenomena. If the irreversible processes are in fact composite, then the laws governing them can only be roughly valid, so to say; they must be of a statistical nature, since they are valid only for a large scale view or for summary consideration; that is, for the average values resulting from a large number of distinct processes. The more we restrict the number of individual events on which these average values are based, the more plainly will occasional divergences from the general or macroscopic law make themselves felt. In other words, if in fact the view described is correct, then the laws of the irreversible processes, like those of friction, heat distribution and diffusion, must without exception be inexact if looked at microscopically; they must admit of exceptions in individual cases; and these exceptions will be the more striking, the more careful our examination becomes.

Now it so turned out in the course of events that experience tended more and more to confirm this conclusion. This could come about, of course, only as the result of a great improvement in the methods of making measurements. The laws governing the irreversible processes come so very near to being absolutely valid because of the enormous number of individual events of which these processes are composed. If, for example, we take a liquid having the same uniform temperature throughout, then it follows by the general or macroscopic law of the conduction of heat that no heat flows within the liquid. Such however is not precisely the case. For heat is the result of slight and rapid movements of the molecules constituting the liquid; the conduction of heat, consequently, is due to the transference of these velocities when the molecules collide. Hence a uniform temperature does not mean that all the velocities are equal, but that the average value of the velocities for each small quantity of liquid is equal. This quantity in fact comprises a large number of molecules. But if we take a quantity containing a relatively small number of molecules, then the average of their velocities will vary; and the variation will be the greater, the smaller is the quantity of liquid. This principle can nowadays be regarded as a fact fully proved by experiment. One of the most striking illustrations is what is known as the Brownian Movement, which can be observed through the microscope in small particles of powder suspended in liquid. These particles are driven backwards and forwards by the invisible molecules of the liquid; the movement is the more pronounced the higher is the temperature. If we make the further assumption, to which in principle there is no objection, that each individual impulse is a reversible event governed by the strict elementary laws of dynamics, then we may say that the introduction of a microscopic method of examination shows that the laws governing the irreversible processes, or what is the same thing, the laws based upon statistics and mere rough approximation, can be traced back to dynamic, accurate, and absolute laws.

The striking results reached by the introduction of statistical laws in many branches of physical research in recent times have produced a remarkable change in the views of physicists. They no longer, as in the earlier days of Energetics, deny or attempt to cast doubt upon the existence of irreversible processes; instead, the attempt is frequently made to place statistical laws in the foreground, and to subordinate to them laws hitherto regarded as dynamic, including even the law of gravitation. In other words, an attempt is made to exclude absolute law from Nature. And indeed, we cannot but be struck by the fact that the natural phenomena which we can investigate and measure can never be expressed by absolutely accurate numbers; for they inevitably contain a certain inaccuracy introduced by the unavoidable defects of measurement itself. Hence it follows that we shall never succeed in determining by measurement whether a natural law is absolutely valid. If we consider the question from the standpoint of the theory of knowledge we come to the same conclusion. For if we cannot even prove that Nature is governed by law (a difficulty which we meet with at the very outset) *a fortiori* we shall be unable to demonstrate that such law is absolute.

Hence from a logical point of view, we must admit every justification for the hypothesis that the only kind of law in Nature is statistical. It is a different question whether this assumption is expedient in physical research; and I feel strongly inclined to answer this question in the negative. We must consider in the first instance that the only type of law fully satisfying our desire for knowledge is the strictly dynamic type, while every statistical law is fundamentally unsatisfactory, for the simple reason that it has no absolute validity but admits of exceptions in certain cases; so that we are continually faced by the question what these particular exceptional cases are.

Questions of this nature constitute the strongest argument in favour of the extension and further refinement of experimental methods. If it is assumed that statistical laws are the ultimate and most profound type in existence, then there is no reason in theory why, when dealing with any particular statistical law, we should ask what are the causes of the variations in the phenomena? Actually, however, the most important advances in the study of atomic processes are due to the attempt to look for a strictly causal and dynamic law behind every statistical law.

On the other hand, we may discover a law which has always proved absolutely valid within the marginal error due to measurement. In such a case we must admit that it will never be possible to prove by means of measurement that it is not after all of the statistical type. At the same time, it is of great importance whether theoretical considerations induce us to regard the law as being of the statistical, or of the dynamic, type. For in the first case, we should attempt to attain the limits of its validity by means of the continuous refinement of our methods of measurement; in the second case, we should regard such attempts as useless and thus save ourselves much unnecessary labour. So much trouble has already been spent in Physics upon the solution of imaginary problems that such considerations are very far from being irrelevant.

In my opinion, therefore, it is essential for the healthy development of Physics that among the postulates of this science we reckon, not merely the existence of law in general, but also the strictly causal character of this law. This has in fact almost universally been the case. Further, I consider it necessary to hold that the goal of investigation has not been reached until each instance of a statistical law has been analysed into one or more dynamic laws. I do not deny that the study of statistical laws is of great practical importance: Physics, no less than meteorology, geography and social science, is frequently compelled to make use of statistical laws. At the same time, however, no one will doubt that the alleged accidental variations of the climatological curves, of population statistics and mortality tables, are in each instance subject to strict causality; similarly, physicists will always admit that such questions are strictly relevant as that which asks why one of two neighbouring atoms of Uranium exploded many millions of years before the other.

All studies dealing with the behaviour of the human mind are equally compelled to assume the existence of strict causality. The opponents of this view have frequently brought forward against it the existence of free will. In fact, however, there is no contradiction here; human free will is perfectly compatible with the universal rule of strict causality—a view which I have had occasion to demonstrate in detail elsewhere. But as my arguments on this subject have been seriously misunderstood in certain quarters, and since this subject is surely of considerable importance, I propose to discuss it briefly here.

The existence of strict causality implies that the actions, the mental processes, and especially the will of every individual are completely determined at any given moment by the state of his mind, taken as a whole, in the previous moment, and by any influences acting upon him coming from the external world. We have no reason whatever for doubting the truth of this assertion. But the question of free will is not concerned with the question whether there is such a definite connection, but whether the person in question is aware of this connection. This, and this alone, determines whether a person can or cannot feel free. If a man were able to forecast his own future solely on the ground of causality, then and then only we would have to deny this consciousness of freedom of the will. Such a contingency is, however, impossible, since it contains a logical contradiction. Complete knowledge implies that the object apprehended is not altered by any events taking place in the knowing subject; and if subject and object are identical this assumption does not apply. To put it more concretely, the knowledge of any motive or of any activity of will is an inner experience, from which a fresh motive may spring; consequently such an awareness increases the number of possible motives. But as soon as this is recognized, the recognition brings about a fresh act of awareness, which in its turn can generate yet another activity of the will. In this way the chain proceeds, without it ever being possible to reach a motive which is definitely decisive for any future action; in other words, to reach an awareness which is not in its turn the occasion of a fresh act of will. When we look back upon a finished action, which we can contemplate as a whole, the case is completely different. Here knowledge no longer influences will, and hence a strictly causal consideration of motives and will is possible, at least in theory.

If these considerations appear unintelligible—if it is thought that a mind could completely grasp the causes of its present state, provided it were intelligent enough—then such an argument is akin to saying that a giant who is big enough to look down on everybody else should be able to look down on himself as well. The fact is that no person, however clever, can derive the decisive motives of his own conscious actions from the causal law alone; he requires another law—the ethical law, for which the highest intelligence and the most subtle self-analysis are no adequate substitute.

§ 7

Let us however return to Physics, from which these complications are excluded in advance. I propose now to describe the more important characteristics of the current view of the physical world. These characteristics are due to the endeavour to find a strict causal connection, in the manner described above, for all physical processes. A cursory glance suffices to show what changes there have been since the beginning of the century; and we may say that since the days of Galileo and Newton, no such rapid development has ever been known. Incidentally we may point with pride to the fact that German scientists have played an important part in this advance. The occasion of this development was that extreme refinement in measurement which is an essential condition of the progress of science and engineering; in its turn this led to the discovery of new facts, and hence to the revision and improvement of theory. Two new ideas in particular have given modern Physics its characteristic shape. These are laid down in the Theory of Relativity and the Quantum hypothesis respectively; each in its own way is at once fruitful and revolutionary; but they have nothing in common and, in a sense, they are even antagonistic.

For a time Relativity was a universal topic of conversation. The arguments for and against could be heard everywhere—even in the daily Press, where it was championed and opposed by experts and by others who were very far from being experts. To-day things have quieted down a little—a state of affairs which is likely to please nobody better than the author of the Theory himself; public interest appears to have become satisfied and to have turned to other popular topics. From this it might perhaps be inferred that the Theory of Relativity no longer plays any part in science. But as far as I can judge, the opposite is the case: for the Theory of Relativity has now become part and parcel of the physical view of the world, and is taken for granted without any further ado. Indeed, novel and revolutionary as was the idea of Relativity (in both the Special and the General form) when first presented to physicists, the fact remains that the assertions it makes and the attacks it delivers were directed not against the outstanding, recognized and approved laws of Physics, but only against certain views which had no better sanction than custom, deeply rooted though they were. These standpoints are of the kind which, as I have already tried to show, afford a suitable basis for a preliminary understanding of the facts of Physics; but they must be discarded as soon as it is found necessary to reach a more general and profound view of the facts.

In this connection the idea of simultaneity is particularly instructive. At first glance, it seems to the observer that nothing could be more obviously true than to say that there is a definite meaning in asersting that two events occurring at two distant points (e.g. on the Earth and on Mars) are simultaneous. Surely every man has a right to traverse great distances timelessly in thought, and to place two events side by side before the mind's eye. Now it must be emphasized that the Theory of Relativity does not alter this right in any way. If we possess sufficiently accurate measuring instruments, we can determine with complete certainty whether the events are simultaneous; and if the time measurements are accurately made in different ways, and with different instruments which can be used to check each other, the same result will always be obtained. To this extent the Theory of Relativity has brought about no change whatever.

But the Theory of Relativity does not allow us to assume, as a matter of course, that another observer who is moving relatively to ourselves must necessarily regard the two events as simultaneous. For the thoughts and ideas of one person are not necessarily the thoughts and ideas of another. If the two observers proceed to discuss their thoughts and ideas, each will appeal to his own measurements; and when they do this, it will be found that in interpreting their respective measurements they started from entirely different assumptions. Which assumption is correct it is impossible to decide; and it is equally impossible to decide the dispute as to which of the two observers is in a state of rest, and which in a state of motion. This question, however, is of fundamental importance. For the rate of a clock alters while the clock is being moved:—a fact which need occasion no surprise; while from this it follows that the clocks of the two observers go at different rates. Thus we reach the conclusion that each can assert with an equal right that he is himself in a state of rest and that his time measurements are correct; and this in spite of the fact that the one observer regards the two events as simultaneous, while the other does not. These ideas and arguments admittedly present a hard task to our powers of imagination; but the sacrifice in clarity is negligible compared with the inestimable advantages which follow from the amazing generality and simplicity of the physical world-view which they render possible.

In spite of this, some readers may still find themselves unable to get rid of the suspicion that the Theory of Relativity contains some kind of internal contradiction. Such readers should reflect that a theory, the entire content of which can be expressed in a single mathematical formula, can no more contain a contradiction than could two distinct conclusions following from the same formula. Our ideas must adjust themselves to the results of the formula and not conversely. Ultimately it is experience that must decide the admissibility and the importance of the Theory of Relativity. Indeed, the fact that experience allows us to test its validity must be looked upon as the most important evidence in favour of the fruitfulness of the theory. Hitherto no instance has been recorded where the Theory conflicts with experience, a fact which I should like to emphasize in view of certain reports which have recently come before the public. Any one who, for whatever reason, considers it possible or probable that a conflict between the Theory and observed facts can be discovered, could do no better than co-operate in extending the Theory of Relativity and in pushing its conclusions as far as possible, since this is the only means of refuting it through experience. Such an undertaking is the less difficult because the assertions made by the Theory of Relativity are simple and comparatively easy to apprehend, so that they fit into the framework of classical Physics without any difficulty.

Indeed, if there were no historical objections I personally would not hesitate for a moment to include the Theory of Relativity within the body of classical Physics. In a manner the Theory of Relativity is the crowning point of Physics, since by merging the ideas of Time and Space it has also succeeded in uniting under a higher point of view such concepts as those of mass, energy, gravitation, and inertia. As the result of this novel view we have the perfectly symmetrical form which the laws of the conservation of energy and of momentum now assume; for these laws follow with equal validity from the Principle of Least Action—that most comprehensive of all physical laws which governs equally mechanics and electrodynamics.

Now over against this strikingly imposing and harmonious structure there stands the Quantum Theory, an extraneous and threatening explosive body which has already succeeded in producing a wide and deep fissure throughout the whole of the structure. Unlike the Theory of Relativity, the Quantum Theory is not complete in itself. It is not a single, harmonious, and perfectly transparent idea, modifying the traditional facts and concepts of Physics by means of a change which, though of the utmost significance in theory, is practically hardly noticeable. On the contrary, it first arose as a means of escape from an *impasse* reached by classical Physics in one particular branch of its studies—the explanation of the laws of radiant heat. It was soon seen, however, that it also solved with ease, or at least considerably helped to elucidate, other problems which were causing unmistakable difficulties to the classical theory, such as photoelectric phenomena, specific heat, ionization, and chemical reactions. Thus it was quickly realized that the Quantum Theory must be regarded, not merely as a working hypothesis, but as a new and fundamental principle of Physics, whose significance becomes evident wherever we are dealing with rapid and subtle phenomena.

Now here we are faced with a difficulty. This does not so much consist in the fact that the Quantum Theory contradicts the traditional views; if that were all, it follows from what has been said that the difficulty need not be taken very seriously. It arises from the fact that in the course of time it has become increasingly obvious that the Quantum Theory unequivocally denies certain fundamental views which are essential to the whole structure of the classical theory. Hence the introduction of the Quantum Theory is not a modification of the classical theory, as is the case with the Theory of Relativity: it is a complete break with the classical theory.

Now if the Quantum Theory were superior or equal to the classical theory at all points, it would be not only feasible but necessary to abandon the latter in favour of the former. This, however, is definitely not the case. For there are parts of Physics, among them the wide region of the phenomena of interference, where the classical theory has proved its validity in every detail, even when subjected to the most delicate measurements; while the Quantum Theory, at least in its present form, is in these respects completely useless. It is not the case that the Quantum Theory cannot be applied, but that, when applied, the results reached do not agree with experience.

The result of this state of affairs is that at the present moment each theory has what may be called its own preserve, where it is safe from attack, while there is also an intermediate region —e.g. that of the phenomena of the dispersion and scattering of light—where the two theories compete with varying fortunes. The two theories are approximately of equal usefulness, so that physicists are guided in the choice of theory by their private predilections—an uncomfortable and, in the long run, an intolerable state of affairs for anyone desirous of reaching the true facts.

To illustrate this curious condition of things I will select a particular example from a very large number collected by workers in the field of theory and of practice. I begin by stating two facts. Let us imagine two fine pencils of rays of violet light, produced by placing an opaque screen with two small holes over against the light which is given out from a point source. The two pencils of rays emerging from the holes can be reflected so that they meet on the surface of a white wall at some distance away. In this case the spot of light which they jointly produce on the wall is not uniformly bright, but is traversed by dark lines. This is the first fact. The second is this—if any metal that is sensitive to light is placed in the path of one of these rays, the metal will continually emit electrons with a velocity independent of the intensity of the light.

Now if the intensity of the source of light is allowed to decrease, then in the first case, according to all the results hitherto obtained, the dark lines remain quite unchanged; it is only the strength of the illumination that decreases. In the other case, however, the velocity of the electrons emitted also remains quite unchanged, and the only change that takes place is that the emission becomes less copious.

Now how do the theories account for these two facts? The first is adequately explained by the classical theory as follows:—at every point of the white wall which is simultaneously illuminated by the two pencils of rays, the two rays which meet at this point either strengthen or else weaken each other, according to the relations between their respective wave-lengths. The second fact is equally satisfactorily explained by the Quantum Theory, which maintains that the energy of the rays falls on the sensitive metal, not in a continuous flow, but in an intermittent succession of more or less numerous, equal and indivisible quanta, and that each quantum, as it impinges on the metal, detaches one electron from the mass. On the other hand, all attempts have failed hitherto to explain the lines of interference by the Quantum Theory and the photoelectric effect by the classical theory. For if the energy radiated really travels only in indivisible quanta, then a quantum emitted from the source of light can pass only through one or else the other of the two holes in the opaque screen; while if the light is sufficiently feeble, it is also impossible for two distinct rays to impinge simultaneously on a single point on the white wall; hence interference becomes impossible. In fact the lines invariably disappear completely, as soon as one of the rays is cut off.

On the other hand, if the energy radiated from a point-source of light spreads out uniformly through space, its intensity must necessarily be diminished. Now it is not easy to see how the velocity with which an electron is emitted from the sensitive metal can be equally great whether it is subjected to very powerful or to very weak radiation. Naturally many attempts have been made to get over this difficulty. Perhaps the most obvious way was to assume that the energy of the electron emitted by the metal is not derived from the radiation falling on it, but that it comes from the interior of the metal, so that the effect of the radiation is merely to set it free in the same way as a spark sets free the latent energy of gunpowder. It has, however, not proved possible to demonstrate that there is such a source of energy, or even to make it appear plausible that there should be such a source. Another supposition is that, while the energy of the electrons is derived from the radiation impinging upon them, the electrons themselves are not actually emitted from the metal until this has been subjected to the illumination for a time sufficiently long to allow the energy necessary for a definite velocity to have been accumulated. This process, however, might take minutes or even hours, whereas in fact the phenomenon repeatedly takes place very much sooner. Light is thrown on the profound importance of these difficulties by the fact that in highly influential quarters the suggestion has arisen of sacrificing the validity of the principle of the conservation of energy. This may well be described as a desperate remedy; in this particular instance, in fact, it was soon proved to be untenable by means of experiments.

Hitherto, then, all attempts to understand the laws of the emission of electrons from the standpoint of the classical theory have failed. On the other hand these, and a number of other laws relating to the reciprocal action of radiation and matter, become immediately intelligible and even necessary as soon as we assume that light quanta travel through space in the shape of minute, individual structures and that, when impinging upon matter, they behave like really substantial atoms.

We are compelled, however, to decide in favour of one or the other view; so that the whole problem obviously resolves itself into the question whether the radiant energy emitted from the source of light is divided when it leaves this source, so that one part of it passes through one of the holes in the opaque screen and the remainder through the other, or whether the energy passes in indivisible quanta alternately through each of the two holes. Every theory of quanta must answer this question, and must deal with it in some manner or other; hitherto, however, no physicist has succeeded in giving a satisfactory answer.

It has sometimes been suggested that the difficulties of the Quantum Theory do not so much apply to the propagation of radiation in free space, as to the reciprocal action which takes place between radiation and matter carrying an electric charge. With this opinion I cannot agree. The question set out above confines itself to the propagation of radiation, and there is no reference either to its causes or to its effects.

It might indeed be asked whether we have a right to speak of the energy of free radiation as though it were something actual, since the fact is that all measurements invariably relate to events taking place in material bodies. If we wish to maintain the absolute validity of the energy principle, a standpoint which recent investigation renders particularly plausible, then there can be no doubt that we must assign to every field of radiation a quite definite, and more or less exactly calculable, amount of energy, which is decreased by the absorption of radiation and increased by its emission. The question now is, what is the behaviour of this energy? Once this question is asked, it becomes plain beyond the possibility of doubt that we must make up our minds to admit certain extensions and generalizations of some of the primary assumptions from which we are accustomed to start in theoretical physics, and which hitherto have proved their worth in every field. This becomes necessary in order to find a way out of the difficulty of our dilemma; and it is a result which is sufficiently unsatisfactory to our desire for knowledge. Some consolation can be derived if we see that there is at any rate a possibility of solving the difficulty; consequently I cannot resist the temptation to devote a few words to discussing in what direction it might be possible to find a solution.

The most radical method of avoiding every difficulty would, no doubt, consist in giving up the customary view which holds that radiant energy is localized in some manner or other; i.e. that at every part of a given electromagnetic field, a given amount of energy exists at a given time. If once this assumption is surrendered, the problem ceases to exist, simply because the question whether a light quantum passes through one or the other hole in the opaque screen ceases to have any definite physical meaning. In my opinion, however, this desperate escape from the dilemma goes somewhat too far. For radiant energy as a totality possesses a definite calculable amount; further, the electromagnetic vector-field which is formed by a ray is described in all its optical details, and in the whole of its temporo-spatial behaviour, by classical electrodynamics, and this description agrees exactly with the facts; finally the energy arises and disappears simultaneously with the field. Consequently it is not easy to avoid the question how the distribution of the energy is affected by the details of the field.

Let us decide to pursue this question as far as possible. Then in order to avoid the alternatives with which we are faced, it might appear expedient to retain the fixed connection between the ray, or rather between the electromagnetic wave on the one hand, and the energy attaching to it on the other, but, while retaining it, to give it a wider and less simple meaning than it has in the classical theory. The latter assumes that every part, however small, of an electromagnetic wave contains a corresponding amount of energy proportional to its magnitude, which is supposed to spread concomitantly with the wave. Now if for this fixed connection we substitute something less rigid, it might then appear that the wave emitted from the source of light divides into any number of parts, in conformity with the classical theory, but that at the same time, in accordance with the Quantum Theory, the energy of the wave is concentrated at certain points. The necessary assumption would be that the energy of the wave is not intimately connected with it in its finest detail. On such an assumption, the phenomena of interference would be explained on the lines that even the weakest wave passes partly through one and partly through the other hole in the opaque screen; while on the other hand the photoelectric effect could be explained on the lines that the wave allows its energy to impinge on the electrons only in integral quanta. Here the difficulty consists in trying to imagine part of a light-wave without the energy appropriate to its magnitude; but though I admit that this is a considerable difficulty, I do not consider it to be essentially greater than that of imagining part of a body without the matter appropriate to its density. Yet we are compelled to make this latter assumption by the fact that matter loses its simple properties if it is subjected to continuous spatial sub-division, since in this case its mass ceases to remain proportional to the space occupied by it, and resolves itself into a number of distinct molecules having a given magnitude. It might well be that the case is closely analogous for electromagnetic energy and the momentum attaching to it.

Hitherto it has been the practice to look for the elementary laws of electromagnetic processes exclusively in the sphere of the infinitely small. Spatially and temporally all electromagnetic fields were divided into infinitely small parts; and their entire behaviour, so far as it appeared subject to laws, was invariably represented by temporo-spatial differential equations. Now in this respect we must radically change our views. For it has been discovered that these simple laws cease to apply after a certain stage in the process of subdivision has been reached, and that beyond this point the increasingly delicate processes make matters more complicated. The spatio-temporal magnitudes of the action become atomic, and we are compelled to assume the existence of elements or atoms of this action. It is indeed a sufficiently striking fact that not a single one of the laws where the universal quantum of action plays a part is expressed by means of a differential equation with a number of continuous variables, but that they all relate to finite times and finite spaces, and deal with such things as definite periods of oscillation, definite orbits, definite transitions, etc. Hence it appears that in order to allow for these facts we must substitute, at least in part, relations between magnitudes at finite distances from each other for those between magnitudes infinitely close to each other. If this is done finite differences take the place of the differential, discontinuity that of continuity, and arithmetic that of analysis; though the substitution admittedly is not carried out radically. A radical substitution is made impossible if only by the claims of the wave theory.

In this direction promising steps have been taken through the development of so-called Quantum Mechanics. This line of investigation has recently produced excellent results in the hands of the Göttingen school of physicists—of Heisenberg, Born and Jordan. Later developments will show how far we can advance towards a solution of the problem along the avenue opened by Quantum Mechanics. Even the choicest mathematical speculations remain in the air so long as they are unsubstantiated by definite facts of experience; and we must hope and trust that the experimental skill of physicists, which in the past has so often definitely decided questions full of doubt and difficulty, will succeed in resolving the difficulties of the present obscure question. In any case there can be no doubt that the parts of the structure of classical Physics, which have had to be discarded as valueless under the pressure of the Quantum Theory, will be supplanted by a sounder and more adequate structure.

To conclude: we have seen that the study of Physics, which a generation ago was one of the oldest and most mature of natural sciences, has to-day entered upon a period of storm and stress which promises to be the most interesting of all. There can be little doubt that in passing through this period we shall be led, not only to the discovery of new natural phenomena, but also to new insight into the secrets of the theory of knowledge. It may be that in the latter field many surprises await us, and that certain views, eclipsed at the moment, may revive and acquire a new significance. For this reason a careful study of the views and ideas of our great philosophers might prove extremely valuable in this direction.

There have been times when science and philosophy were alien, if not actually antagonistic to each other. These times have passed. Philosophers have realized that they have no right to dictate to scientists their aims and the methods for attaining them; and scientists have learned that the starting-point of their investigations does not lie solely in the perceptions of the senses, and that science cannot exist without some small portion of metaphysics. Modern Physics impresses us particularly with the truth of the old doctrine which teaches that there are realities existing apart from our sense-perceptions, and that there are problems and conflicts where these realities are of greater value for us than the richest treasures of the world of experience.

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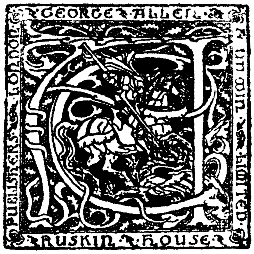

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